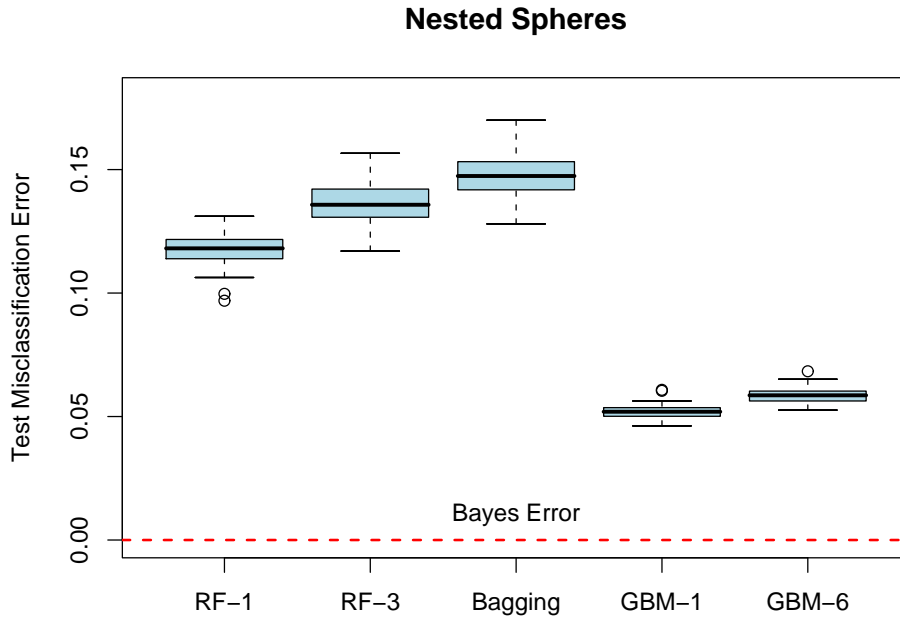
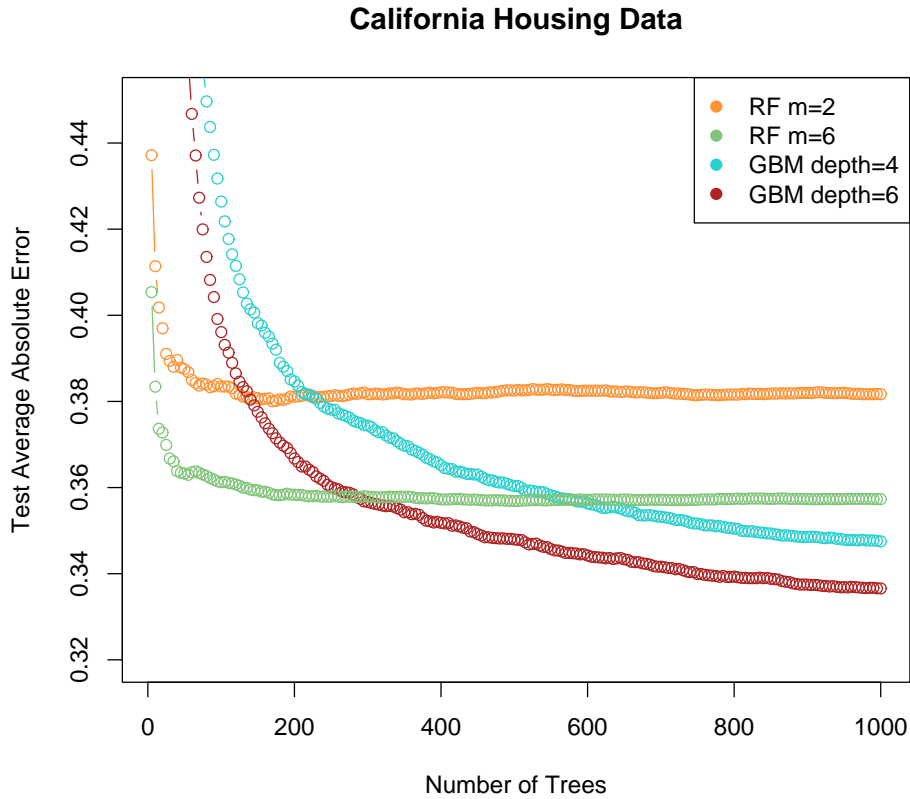


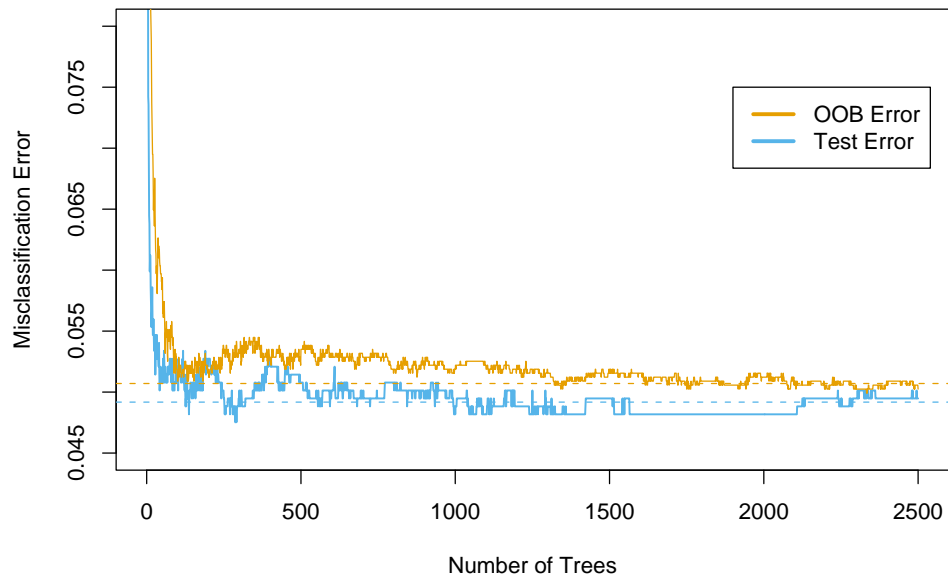
**FIGURE 15.1.** *Bagging, random forest, and gradient boosting, applied to the spam data. For boosting, 5-node trees were used, and the number of trees were chosen by 10-fold cross-validation (2500 trees). Each “step” in the figure corresponds to a change in a single misclassification (in a test set of 1536).*



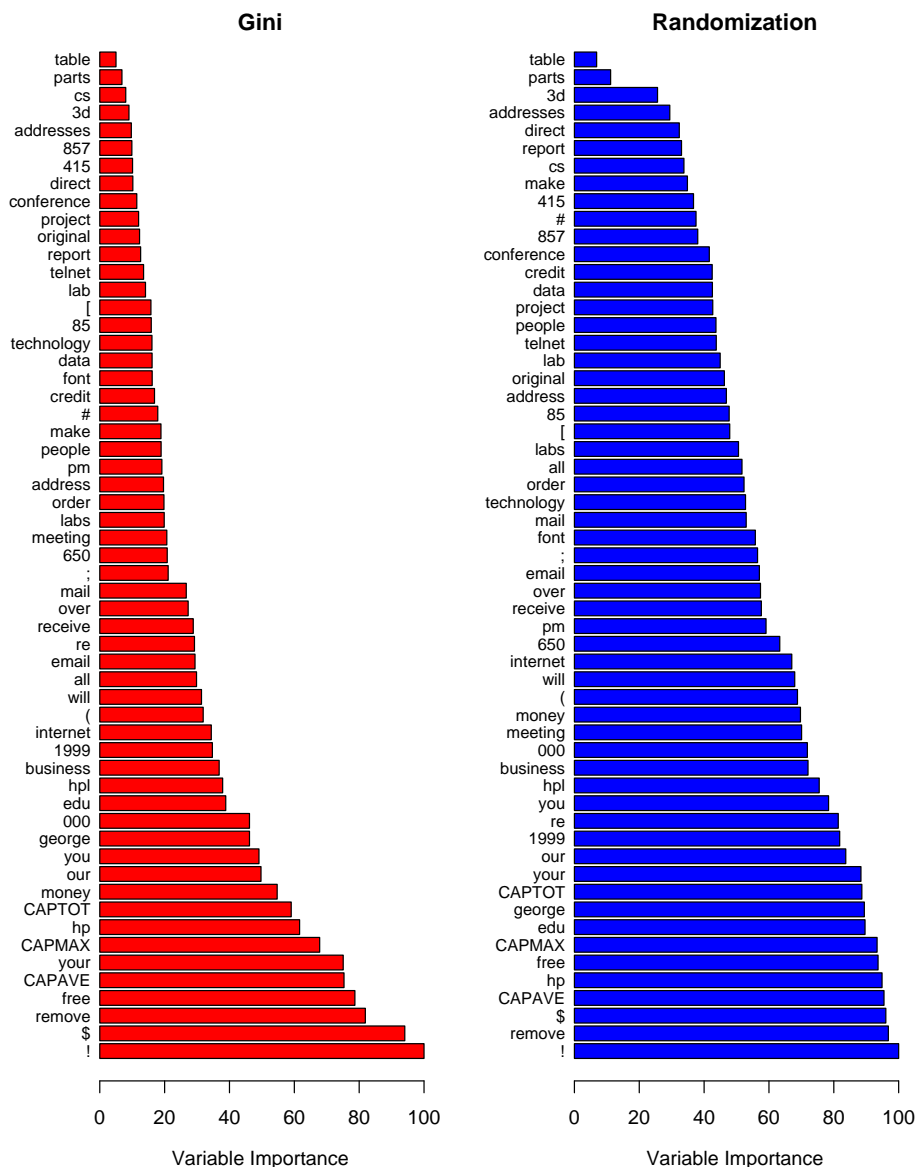
**FIGURE 15.2.** *The results of 50 simulations from the “nested spheres” model in  $\mathbb{R}^{10}$ . The Bayes decision boundary is the surface of a sphere (additive). “RF-3” refers to a random forest with  $m = 3$ , and “GBM-6” a gradient boosted model with interaction order six; similarly for “RF-1” and “GBM-1.” The training sets were of size 2000, and the test sets 10,000.*



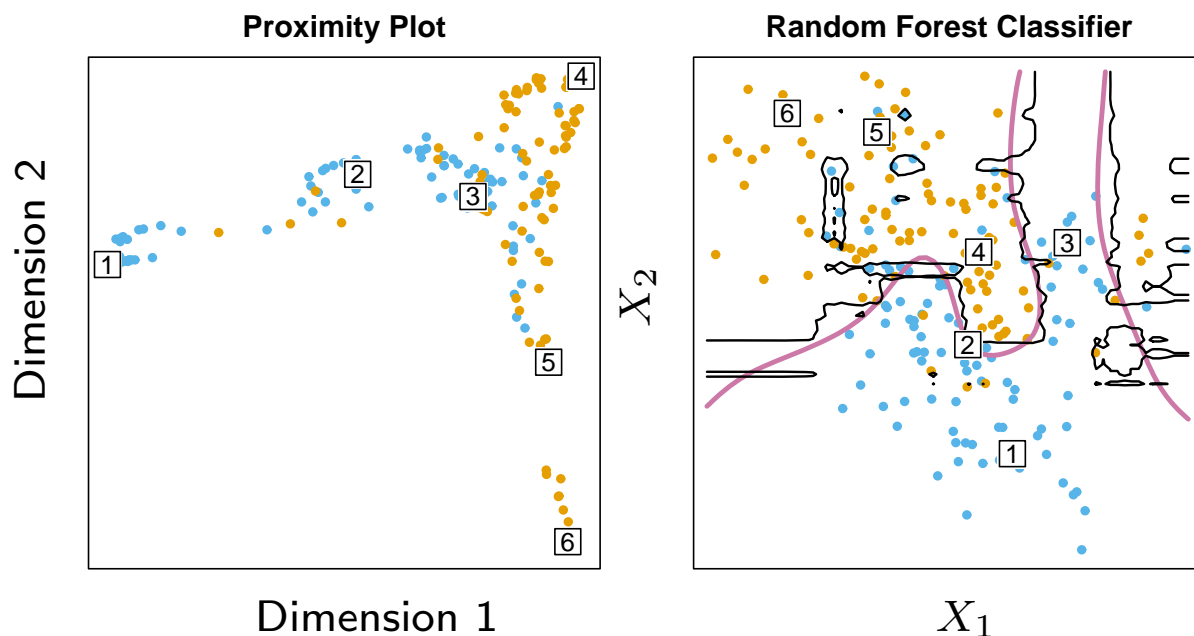
**FIGURE 15.3.** Random forests compared to gradient boosting on the California housing data. The curves represent mean absolute error on the test data as a function of the number of trees in the models. Two random forests are shown, with  $m = 2$  and  $m = 6$ . The two gradient boosted models use a shrinkage parameter  $\nu = 0.05$  in (10.41), and have interaction depths of 4 and 6. The boosted models outperform random forests.



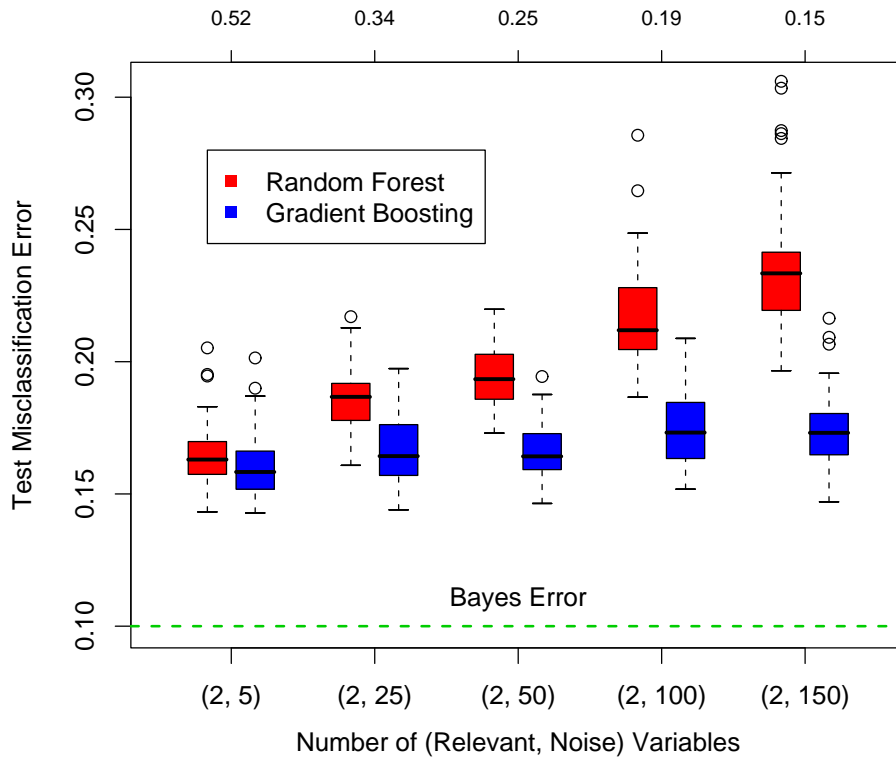
**FIGURE 15.4.** OOB error computed on the spam training data, compared to the test error computed on the test set.



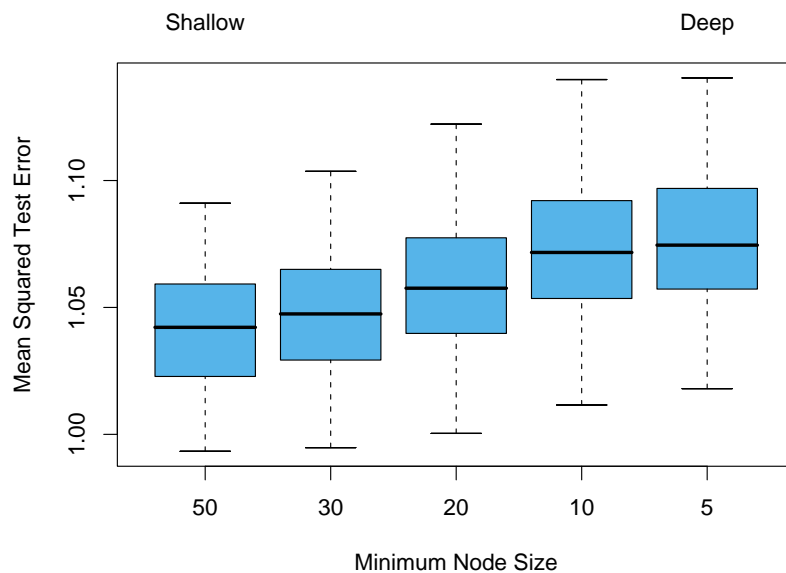
**FIGURE 15.5.** Variable importance plots for a classification random forest grown on the `spam` data. The left plot bases the importance on the Gini splitting index, as in gradient boosting. The rankings compare well with the rankings produced by gradient boosting (Figure 10.6 on page 316). The right plot uses OOB randomization to compute variable importances, and tends to spread the importances more uniformly.



**FIGURE 15.6.** (Left): Proximity plot for a random forest classifier grown to the mixture data. (Right): Decision boundary and training data for random forest on mixture data. Six points have been identified in each plot.

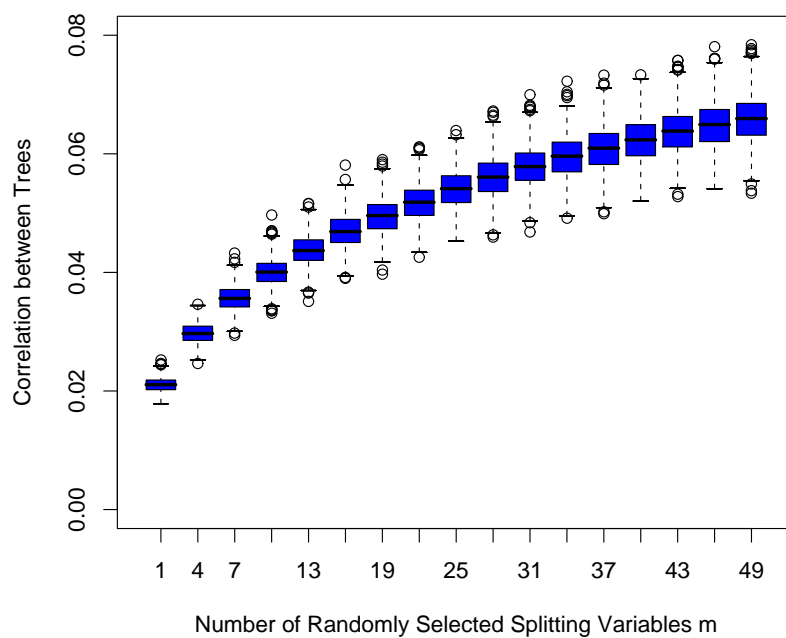


**FIGURE 15.7.** A comparison of random forests and gradient boosting on problems with increasing numbers of noise variables. In each case the true decision boundary depends on two variables, and an increasing number of noise variables are included. Random forests uses its default value  $m = \sqrt{p}$ . At the top of each pair is the probability that one of the relevant variables is chosen at any split. The results are based on 50 simulations for each pair, with a training sample of 300, and a test sample of 500.

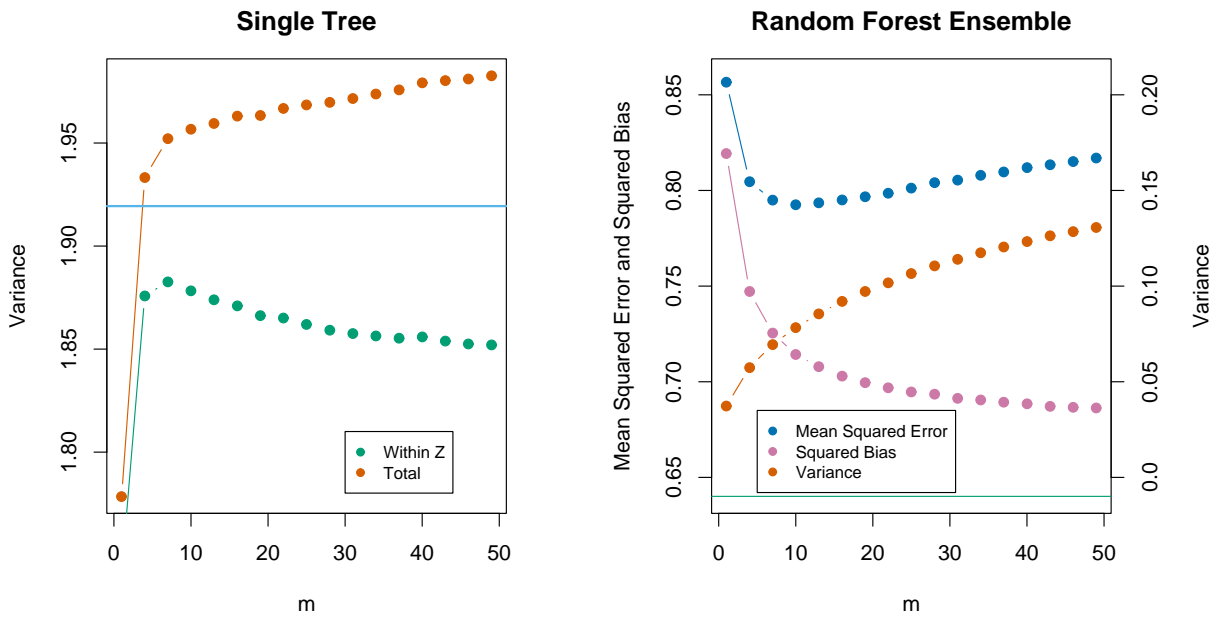


**FIGURE 15.8.** *The effect of tree size on the error in random forest regression. In this example, the true surface was additive in two of the 12 variables, plus additive unit-variance Gaussian noise. Tree depth is controlled here by the minimum node size; the smaller the minimum node size, the deeper the trees.*

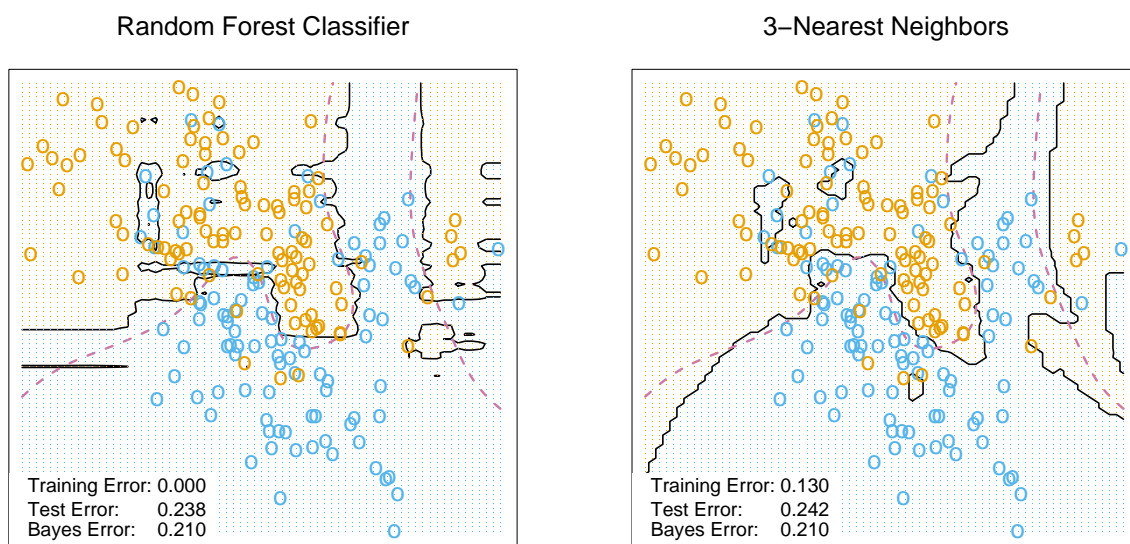




**FIGURE 15.9.** *Correlations between pairs of trees drawn by a random-forest regression algorithm, as a function of  $m$ . The boxplots represent the correlations at 600 randomly chosen prediction points  $x$ .*



**FIGURE 15.10.** *Simulation results. The left panel shows the average variance of a single random forest tree, as a function of  $m$ . “Within  $\mathbf{Z}$ ” refers to the average within-sample contribution to the variance, resulting from the bootstrap sampling and split-variable sampling (17.9). “Total” includes the sampling variability of  $\mathbf{Z}$ . The horizontal line is the average variance of a single fully grown tree (without bootstrap sampling). The right panel shows the average mean-squared error, squared bias and variance of the ensemble, as a function of  $m$ . Note that the variance axis is on the right (same scale, different level). The horizontal line is the average squared-bias of a fully grown tree.*



**FIGURE 15.11.** *Random forests versus 3-NN on the mixture data. The axis-oriented nature of the individual trees in a random forest lead to decision regions with an axis-oriented flavor.*